# *Golomb Patterns: Introduction, Applications, and Citizen Science Game*

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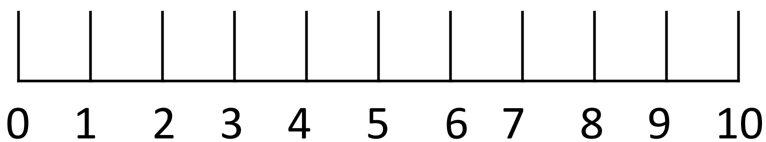
# Talk Overview

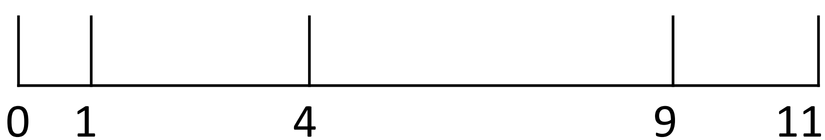
- Introduction to Golomb Rulers
  - Definition
  - Properties and variants
  - Known results
- Applications
- Our Past work
  - Results
  - Application example
    - Astrophysics
    - EPIC mission
- Current Work
  - Citizen science game design
  - Development status
- Summary

# Golomb Rulers

- A *Golomb ruler* is a ruler with marks at integer positions such that pair-wise non-zero distances between its marks are all distinct.



A typical ruler



A Golomb ruler

Pair wise distance between marks

| | **0** | **1** | **4** | **9** | **11** |
|---|---|---|---|---|---|
| **0** | 0 | 1 | 4 | 9 | 11 |
| **1** | 1 | 0 | 3 | 8 | 10 |
| **4** | 4 | 3 | 0 | 5 | 7 |
| **9** | 9 | 8 | 5 | 0 | 2 |
| **11** | 11 | 10 | 7 | 2 | 0 |

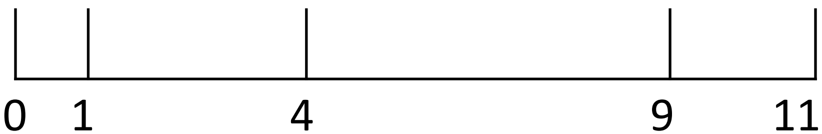
# Golomb Rulers …

- A set of integers A = $\{a_1, a_2, \ldots, a_n\}$,
  - $a_1 < a_2 < \ldots < a_n$
  - $\binom{n}{2}$ Differences $\{a_i - a_j \mid 1 \leq i < j \leq k\}$ are distinct.
  - Ruler has order $n$ and length $l = a_n - a_1$ .
  - For simplicity, let $a_1 = 0$ and $a_n = l$ .

- Equivalent to *Sidon sets* problem:
  - Set of ordered natural numbers, such that their sums are different.

# Golomb Rulers

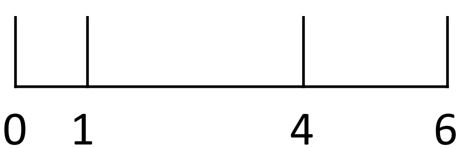
- *Optimal Golomb ruler* of order $n$ is a Golomb ruler for which $a_n$ is the smallest.




A Golomb ruler

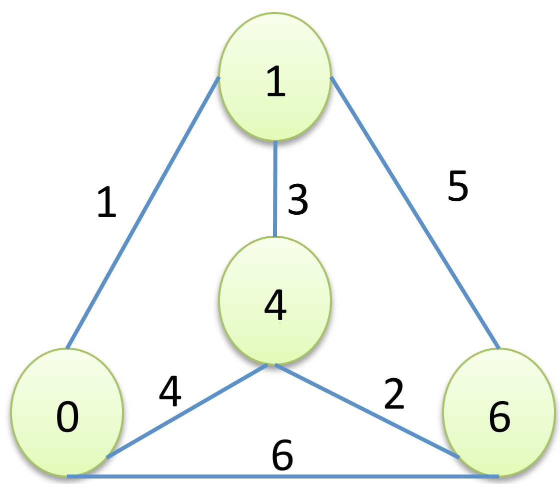
- A Golomb ruler is a *perfect* Golomb ruler if

$| a_i - a_j |$ are consecutive integers from *0* to *l* .




A perfect Golomb ruler

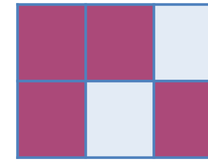
| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 0 | 1 |



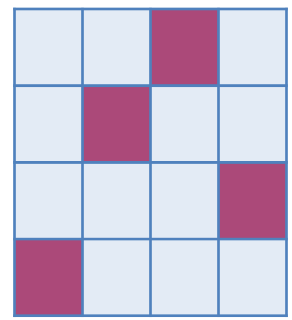

Distances between marks

| | **0** | **1** | **4** | **6** |
|---|---|---|---|---|
| **0** | 0 | 1 | 4 | 6 |
| **1** | 1 | 0 | 3 | 5 |
| **4** | 4 | 3 | 0 | 2 |
| **6** | 6 | 5 | 2 | 0 |

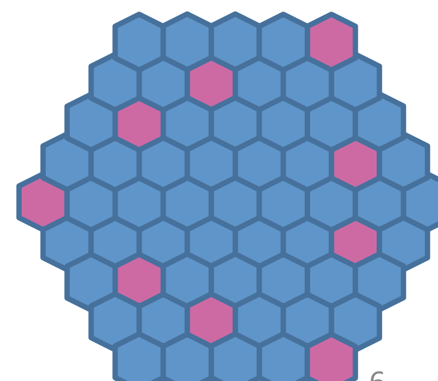
# Golomb Ruler Variants



- 2D case: Golomb rectangles
  - N x M matrix of 0s and 1s
  - Autocorrelation values 0, or 1, or k (the number of ones in the rectangle).
  - 1-1 relationship between a Golomb ruler and rectangle
    - N x M Golomb rectangle with k ones
    - K element Golomb ruler from the set $\{i + (2N-1)(j-1) \mid 1 \leq i \leq N,\ 1 \leq j \leq M\}$



- Costas arrays
  - 1960s by John Costas
  - A Golomb rectangle
  - Exactly one 1 in each row and column



- Honeycomb arrays
  - A Costas array in the hexagonal grid
  - Introduced by Golomb and Taylor in 1984
  - Three different row directions
  - Exactly one 1 in each row.

# Known Results

- Conjectured to be NP-hard
  - – Exponential time search
- Optimal rulers
  - – longest of length 492 with 26 marks.
- Optimal rectangles
  - – Rectangles with 1…17 marks.
    - $10 \times 14$ rectangle with 17 marks
- No perfect Golomb ruler with *more than four marks.*
  - Suppose there exists one with 5 marks.
- Costas arrays
  - – Solutions for all $n \leq 29$.
  - – No Costas arrays have been found for $n = 32$ or $n = 33$
  - – Do Costas arrays exist for any n?

# Applications

- Coding Theory (Kautz, 1954)
  - Increased error detection and correction by lowering information transmission rate
- Sensor placement in X-Ray Crystallography
  - Distinguish different crystal structures that have identical X-ray diffraction patterns
  - A crystal structure is unique, if all diffraction patterns are congruent.
- Radio-frequency allocation (Babock 1953)
- Communication Network labeling
  - Assign each user a terminal node number
- Circuit Layout
- Antenna design for Radar missions
- Sonar Applications
  - Costas was a U.S. navy radar engineer during WWII.
  - *Music!*
    - *Rickard's TED talk: "The beautiful math behind the ugliest music".* http://www.ted.com/talks/scott_rickard_the_beautiful_math_behind_the_ugliest_music.html
- *Linear telescope arrays in radio astronomy* (Blum 1974)

# Applications in Astrophysics and Earth Sciences

- Motivation:
  - Large telescopes with high angular resolution in far infrared
    - Resolution similar to Hubble, need a 1km diameter telescope
  - Impractical and expensive to launch
  - Similarly for large antennas in Radar for Earth Sciences
    - Multiple small apertures (Synthetic Aperture Radar)

- Solution:
  - Multiple small apertures
  - Combined with interferometry techniques

- Computationally hard
  - Exponential number of possible configurations
  - Each Aperture should provide unique (non-redundant) information

# Objective

- Find optimal number of Non-Redundant Apertures (NRAs) that maximizes the *fill factor* of a grid.
  – *Fill factor*, for a 2D grid with a non-redundant aperture pattern, is the percentage of non-zero autocorrelation values for that pattern to total possible number of auto-correlation values.

- Develop a Non-Redundant Aperture Solver and Visualizer tool for astrophysicists. Interferometry missions can use this tool for planning
    - Number of apertures and their positions
    - Studying science vs. cost trade-offs of each configuration.

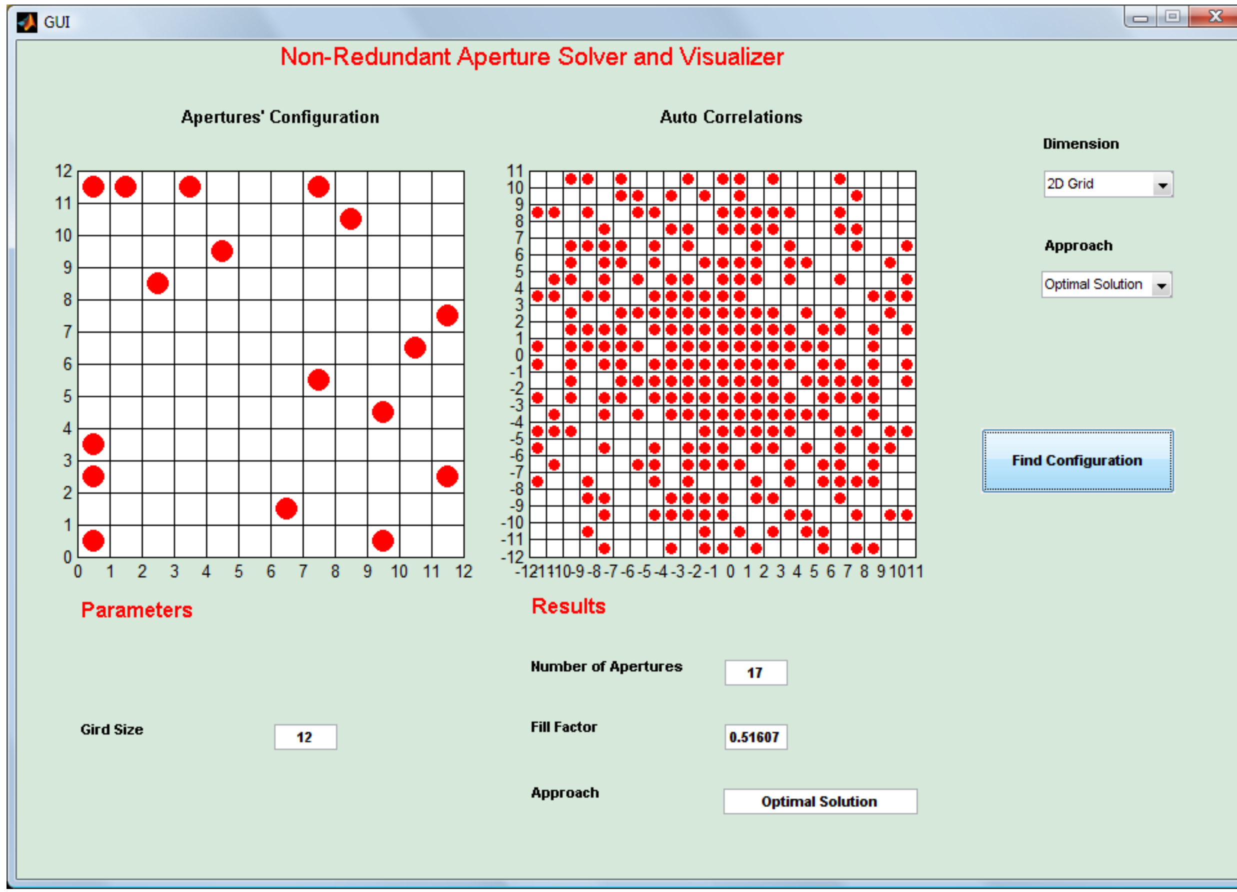
- Propose solutions for the Extrasolar Planetary Imaging Coronagraph Imaging (EPIC) mission.

# Our Past Results…

## 2009-2010 IRAD with Rick Lyon and David Mount

We designed and developed a Matlab Graphical User Interface (GUI) that

- Accepts user's input on grid size and desired algorithm to use.
- Interfaces with our Matlab routines to find a solution.
- Displays the NRA pattern found (left), as well as it's auto-correlation matrix (right).
- Reports the fill-factor and number of apertures found.



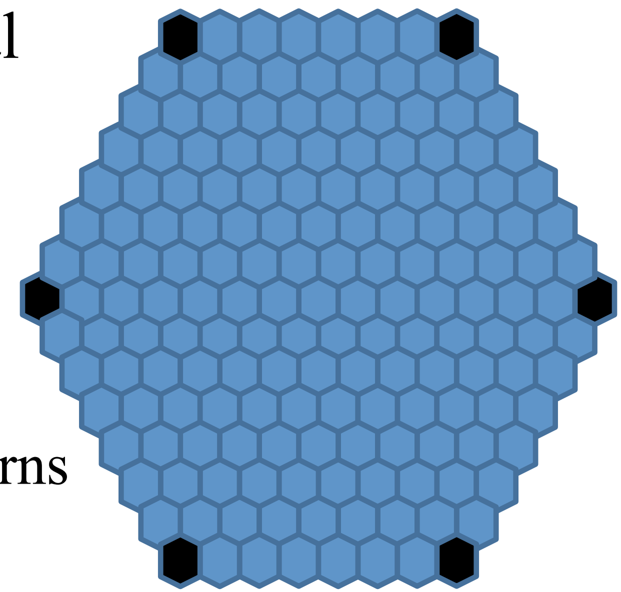

The Visualizer showing a 12 x 12 optimal NRA. The left hand side grid shows the NRA pattern, while on the right hand side we see the corresponding uv-plane coverage.

# Our Past Results

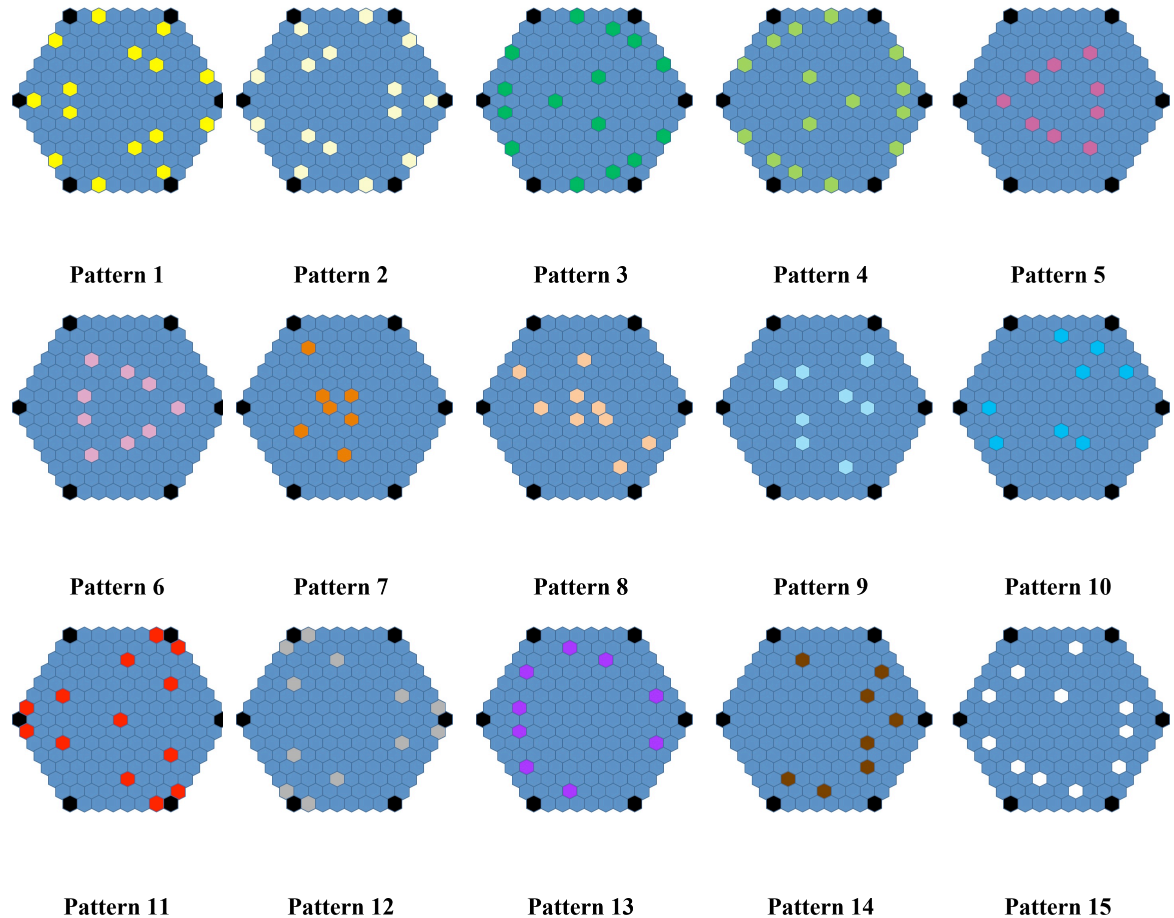
- Proved that there are no *perfect* Golomb rectangles (those with a 100% fill factor).
- Proved that the fill factor of Costas arrays converges to 25% as grid's dimensions increase.
- Developed a Matlab package solving NRA via different approaches
    - Welch, Golomb, Lempel, Taylor, Ruzsa, and search algorithms.
- Compared the fill factor of the results.
- Developed a graphical user interface in Matlab for visualization of the results of various statistics of interest.
- Find NRAs in 2D by solving an equivalent problem in 1D
- Improved past results used for Stellar Imager (SI).
- Can similarly improve the science return of other interferometry missions for the same cost.
- Scientists can use the developed Matlab package through an easy-to-use interface when planning and estimating the cost of the interferometry missions.
- Users can interface with other algorithms and can visualize the results

# Astrophysics Application: EPIC mission

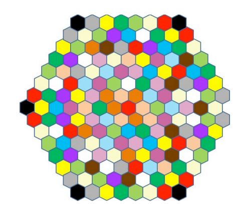
- Extrasolar Planetary Imaging Coronagraph (EPIC) mission
- Given a honeycomb array of 169 hexagonal cells
  - MEMS based deformable mirror (DM)
  - 163 of them are active
  - 6 of them are not active
- *Identify sets of active DMs such that*
  - They each form non-redundant aperture patterns
  - Their union be all blue cells
  - Their intersection be empty
- *Approach:*
  - Use the known honeycomb patterns of radius 1, 3, 4, and 7 and their rotations



Future developments may include support for hexagonal Golomb arrays. Applications of such patterns are interferometry missions such as EPIC using Visible Nulling Coronagraph that consists of deformable mirrors (DMs) of hexagonal shape.



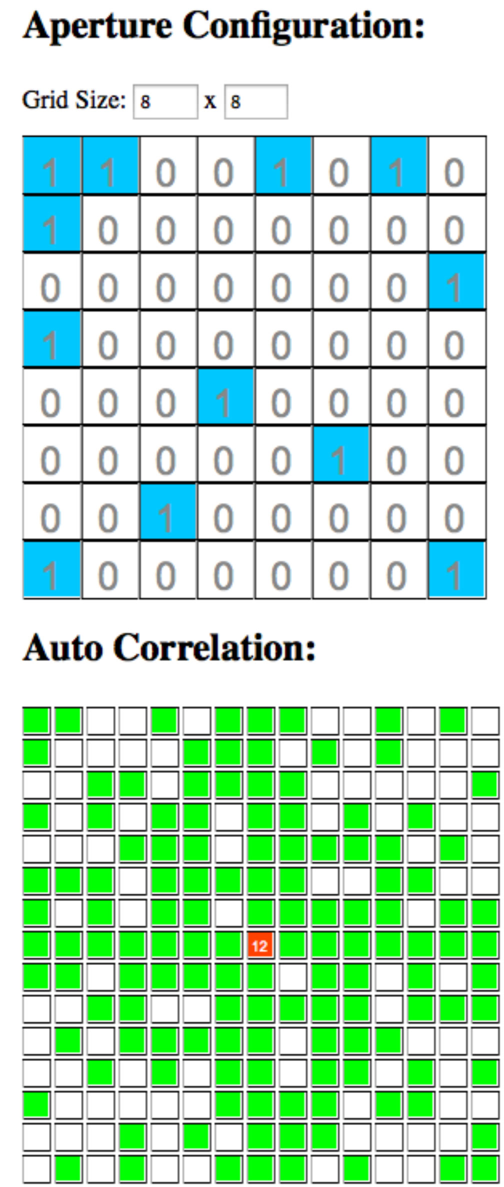

Visible Nulling Coronagraph (VNC) consists of 169 hexagonal deformable mirrors (DMs), six of which are not active (black cells). We want to decompose the remaining active DMs into non-redundant aperture patterns such that 1) each pattern be a non-redundant Golomb pattern on a hexagonal grid, 2) The intersection of the set of mirrors in each pattern be empty, and 3) the union of the set of mirrors in the recommended patterns be the set of all 163 active mirrors. Recommended solution (configuration A) was achieved by ad-hoc modifications and improvements made to the already published Honeycomb array solutions (S. Blackburn, 2010) with radius 1, 3, 5, and 7 (Patterns 1-15).



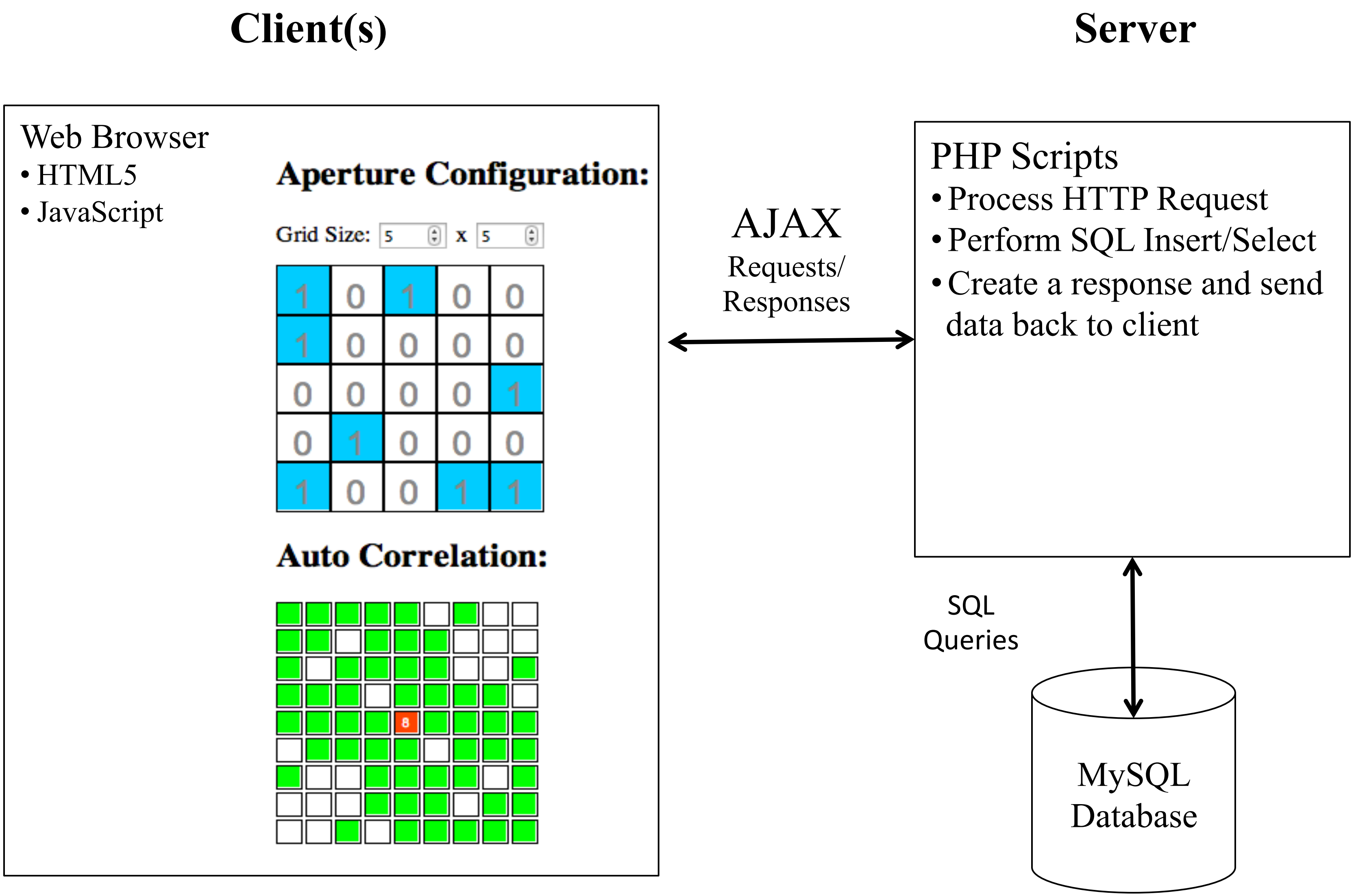
# Game and Repository for Aperture Solutions and Patterns (GRASP)

*Citizen Science Game to solve computationally hard problems*

- Team
  - Nargess Memarsadeghi, Principal Investigator
  - Jeffrey Hostler, Lead Developer
  - Richard G. Lyon, Project co-Investigator
  - Marc Kuchner, Proposal Consultant
- Funding Source: NASA Goddard's Science Innovation Fund (SIF)
- Funding Period: January-October, 2013
- Objective
  - Engage public to find (optimal) non-redundant (aperture) patterns
  - Develop interactive and thought provoking game to solve problem
  - Computationally hard due to exponential number of possibilities
    - *Humans can often solve through cognitive, yet little understood, processes.*



# Citizen Science Game Design



# Potential Impacts

- Finding new solutions by engaging the general public
  - Thousands of users from all over the world
  - Growing number of known patterns
  - Growing number of students in STEM fields
- A resource and tool for scientists and engineers
  - Contributing to new results
  - Repository of Golomb Patterns
  - Visualizing solutions and their properties
    - Mission studies
    - Planning

# Summary

- Finding optimal Golomb rulers and rectangles is a computationally hard problem.
- Reviewed some past work and results.
- Applications in various fields such as coding theory, communications, astronomy, earth, and planetary sciences.
- Citizen Science Game: GRASP!

# To learn more …

- Please visit: Educational NASA Computational and Scientific Studies (*enCOMPASS*) project:
  - http://encompass.gsfc.nasa.gov
  - NASA Computational Case Studies in
    - Earth Sciences
    - Planetary Sciences
    - Astrophysics
  - Please stay tuned for the release of the GRASP and a related case study on the enCOMPASS site soon!
  - Please sign up to receive updates on this project!

# References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- Matlab Toolbox, solutions, and papers by Scott Rickard *et. al.* http://www.costasarrays.org/
- "The beautiful math behind the ugliest music" by Scott Rickard: *http://www.ted.com/talks/scott_rickard_the_beautiful_math_behind_the_ugliest_music.html*
- http://www.research.ibm.com/people/s/shearer/golrec.html

# Questions?

*Thank You!*